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Complex Nos.

* A no. 'z' of the form $z = x + yi$ is known as complex no. where $Re(z) = x$ and $Im(z) = y$

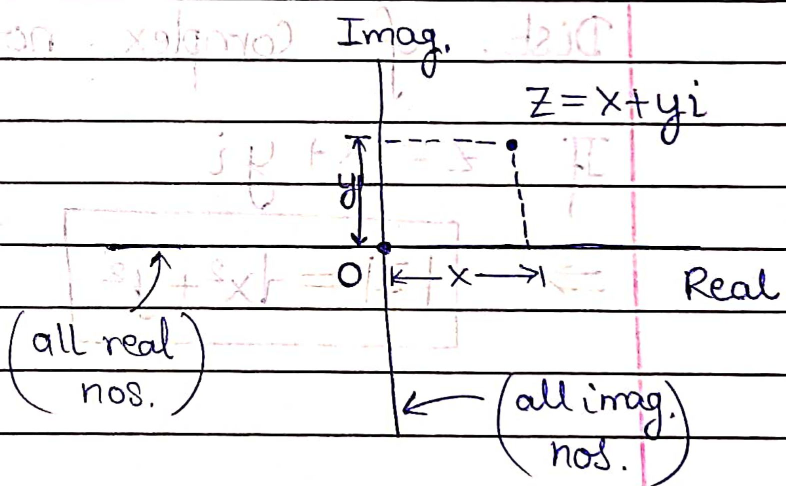
$$(x + yi) \equiv \text{Ordered pair } (x, y)$$

where $\boxed{i = \sqrt{-1}}$ is the most fundamental complex no.

* $i^2 = (-1)$, $i^3 = (-i)$, $i^4 = 1$

* Representation —

In Argand plane,



☆ 0 is Purely Real as well as Purely Imaginary.

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Algebraic Operations

Let $z_1 = (x_1 + y_1 i)$, $z_2 = (x_2 + y_2 i)$.

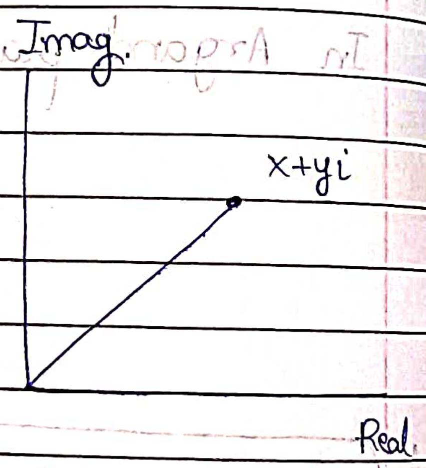
- $(z_1 \pm z_2) = (x_1 \pm x_2) + (y_1 \pm y_2) i$
- $(z_1 z_2) = (x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1) i$
- $\left(\dfrac{z_1}{z_2}\right) = \dfrac{(x_1 x_2 + y_1 y_2)}{(x_2^2 + y_2^2)} + \dfrac{(x_2 y_1 - x_1 y_2)}{(x_2^2 + y_2^2)} i$

Modulus

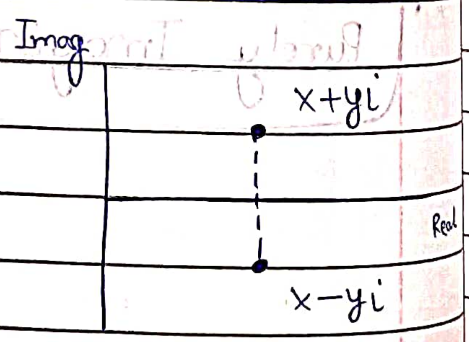
Dist. of complex no. from O.

If $z = x + yi$

$\Rightarrow \boxed{|z| = \sqrt{x^2 + y^2}}$



Conjugate



$z = x + yi$; $\overline{z} = x - yi$

$(z_1\bar{z}_2 + \bar{z}_1 z_2)$ is ALWAYS Real

Image of ~~Real~~ Complex no. in Real axis, is called its Conjugate.

Props of Modulus & Conjugate

- $|z| = |\bar{z}|$
- $(z + \bar{z}) = 2\,\mathrm{Re}(z)$
- $(z - \bar{z}) = 2\,\mathrm{Im}(z)$

☆ $\boxed{z\bar{z} = |z|^2}$

- $\overline{(z_1 + z_2)} = \overline{(z_1)} + \overline{(z_2)}$

Generally, $\overline{(z_1 + \dots + z_n)} = \overline{(z_1)} + \dots + \overline{(z_n)}$

- $\overline{(z_1 z_2)} = \overline{(z_1)}\,\overline{(z_2)}$

Generally, $\overline{(z_1 \dots z_n)} = \overline{(z_1)} \dots \overline{(z_n)}$

- $\overline{(z^n)} = (\bar{z})^n$
- $\overline{(\bar{z})} = z$
- $|z| \iff z = 0 + 0i$
- $-|z| \leq \mathrm{Re}(z) \leq |z|$
  $-|z| \leq \mathrm{Im}(z) \leq |z|$
- $|z_1 \dots z_n| = |z_1| \dots |z_n|$

☆ z purely Real $\iff z = \bar{z}$
z purely Imag. $\iff (z + \bar{z}) = 0$

Imp. Inequality —

$(= \text{ if origin int. divides } z_1 \,\&\, z_2)$

$$\boxed{\big||z_1| - |z_2|\big| \leq |z_1 \pm z_2| \leq |z_1| + |z_2|}$$

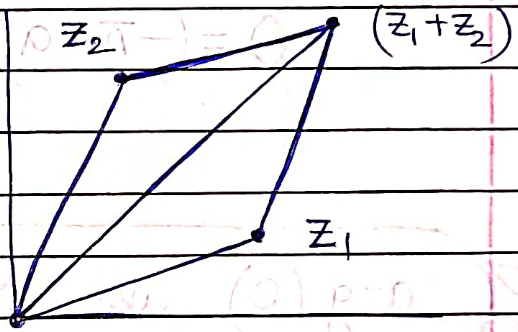
$(= \text{ if origin ext. divides } z_1 \,\&\, z_2.)$

$\forall \ z_1, z_2 \in \mathbb{C}$

→ In general, $|z_1 + \ldots + z_2| \leq |z_1| + \ldots + |z_2|$

Geometrical Meaning :

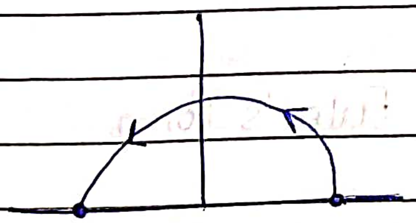
- Mark pts z_1, z_2.
- Complete $\|$gm.
- Above inequality is same as Δ inequality.



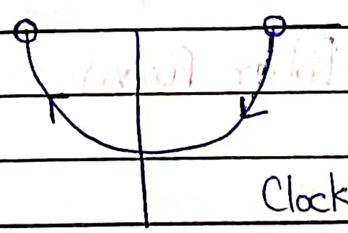
Argument —

Solution of eqⁿs $\boxed{\cos(\theta) = \frac{x}{|z|}}$ & $\boxed{\sin(\theta) = \frac{y}{|z|}}$

Principal Arg. : $\boxed{(-\pi) < \theta \leq \pi}$



Anticlockwise



Clockwise

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For finding principal arg.,

$$\boxed{\alpha = \tan^{-1}\left(\left|\frac{y}{x}\right|\right)}$$

Then plot on argand plane,

$\theta = (\pi - \alpha)$	$\theta = \alpha$
$\theta = (-\pi + \alpha)$	$\theta = (-\alpha)$

☆ arg(0) is NOT defined.

Euler's Form & Polar Form —

Let $z = x + yi$. If $|z| = r$ & $\arg(z) = \theta$.

$$\boxed{z = r(\cos(\theta) + i\sin(\theta))} \; ; \; \boxed{z = re^{i\theta}}$$

Polar form

Euler's form

$$\boxed{e^{i\theta} = \cos(\theta) + i\sin(\theta)}$$

Whenever we have to assume a complex no. with unit modulus, we use this

Props of Arg. —

z_1	z_2	n
-1	-1	1
i	-1	-1

Examples to verify

- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2n\pi$; $n \in \{-1, 0, 1\}$
- $\arg(z) = 0, \pi \iff z$ is Purely Real.
- $\arg(z) = (-\pi/2), (\pi/2) \iff z$ is Purely Imag.
- $\arg(z) + \arg(\bar{z}) = 0$.

Q) If $\left|z - \frac{1}{z}\right| = 1$, find $|z|_{max}$ & $|z|_{min}$.

Q) P.t. $\nexists z \in \mathbb{C}$ s.t. $|z| < 1/3$ and $\sum_{r=1}^{\infty}(a_r z^r) = 1$ and $|a_r| < 2$

Q) If $z \in \mathbb{C}$ and $z^2 - az + b = 0$ $(a \neq 0)$ has 2 roots of unit modulus, then p.t.

1) $|a| \leqslant 2$ 2) $|b| = 1$ 3) $\arg(b) = 2\arg(a)$.

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A) $\left|z-\frac{1}{z}\right|=1 \Rightarrow \left||z|-\frac{1}{|z|}\right| \leq 1$

$\Rightarrow |z|^2-|z|-1 \leq 0$ & $|z|^2+|z|-1 \geq 0$

$\Rightarrow |z| \in \left[\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right]$ & $z \notin \left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$

$\Rightarrow \boxed{|z| \in \left[\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right]}$

A) Let us assume for sake of contradiction, there exists such a no.

$$\left|\sum_{r=1}^{\infty}(a_r z^r)\right| \leq \sum_{r=1}^{\infty}\left(|a_r||z|^r\right) < 2\sum_{r=1}^{\infty}\left(|z|^r\right)$$

$$= \frac{2|z|}{1-|z|}$$

Now $|z| < 1/3 \Rightarrow (1-|z|) > \frac{2}{3} \Rightarrow \left(\frac{1}{1-|z|}\right) < \frac{3}{2}$

$\Rightarrow \left(\frac{|z|}{1-|z|}\right) < \frac{1}{2}$

$\Rightarrow \left|\sum_{r=1}^{\infty}(a_r z^r)\right| < 1 \Rightarrow$ Contradiction !

A) 1) $|a| = |z_1 + z_2| \leq |z_1| + |z_2| \Rightarrow \boxed{|a| \leq 2}$

2) $|b| = |z_1 z_2| \Rightarrow \boxed{|b| = 1}$

3) Let $z_1 = \cancel{e^{i\theta}}(c_\theta + i s_\theta)$; $z_2 = (c_\phi + i s_\phi)$

$\arg(b) = \arg(c_{\theta+\phi} + i s_{\theta+\phi}) = (\theta + \phi)$

$\arg(a) = \arg\left((c_\theta + i s_\theta) + (c_\phi + i s_\phi)\right) = \arg\left(2 c_{\left(\frac{\theta-\phi}{2}\right)}\left(c_{\left(\frac{\theta+\phi}{2}\right)} + i\, s_{\left(\frac{\theta+\phi}{2}\right)}\right)\right)$

$\Rightarrow \arg(a) = \left(\frac{\theta+\phi}{2}\right) \Rightarrow \boxed{\arg(b) = 2\arg(a)}$

★ Alternate Sol^n : $\arg(z_1 + z_2) = \arg(z_1 z_2) + \arg\left(\frac{1}{z_1} + \frac{1}{z_2}\right)$

$= \arg(z_1 z_2) + \arg(\bar{z}_1 + \bar{z}_2) = \arg(z_1 z_2) - \arg(z_1 + z_2)$

$\Rightarrow 2\arg(z_1 + z_2) = \arg(z_1 z_2)$

$\Rightarrow \boxed{2\arg(a) = \arg(b)}$

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Q) If $|z| \leq 1$ & $|w| \leq 1$, then p.t.

$$|z-w|^2 \leq (|z|-|w|)^2 + (\arg(z) - \arg(w))^2$$

A) Let $z = r_1 e^{i\theta_1}$ & $w = r_2 e^{i\theta_2}$.

- $|z-w|^2 = (r_1 c_{\theta_1} - r_2 c_{\theta_2})^2 + (r_1 s_{\theta_1} - r_2 s_{\theta_2})^2$

$= r_1^2 + r_2^2 - 2r_1 r_2 c_{\theta_1-\theta_2}$

- $(|z|-|w|)^2 = r_1^2 + r_2^2 - 2r_1 r_2$
- $(\arg(z) - \arg(w))^2 = (\theta_1 - \theta_2)^2$

Now, $\boxed{(|z|-|w|)^2 + (\arg(z) - \arg(w))^2}$

$= r_1^2 + r_2^2 - 2r_1 r_2 + (\theta_1 - \theta_2)^2$

$= (r_1^2 + r_2^2 - 2r_1 r_2 c_{\theta_1-\theta_2}) + 2r_1 r_2 (c_{\theta_1-\theta_2} - 1) + (\theta_1 - \theta_2)^2$

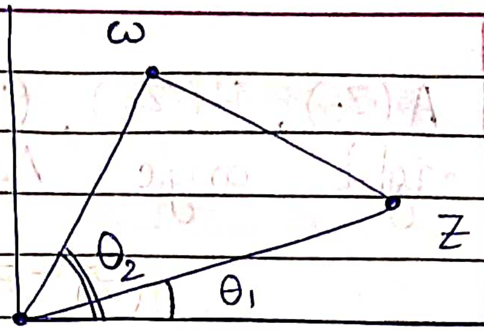
$= \boxed{|z-w|^2 + (\theta_1 - \theta_2)^2 - 4r_1 r_2 \sin^2\left(\frac{\theta_1-\theta_2}{2}\right)}$

Now, $4r_1 r_2 \sin^2\left(\frac{\theta_1-\theta_2}{2}\right) \leq 4(1)(1)\left(\frac{\theta_1-\theta_2}{2}\right)^2 = (\theta_1-\theta_2)^2$

$\Rightarrow$ Req. is proven.

Better Method :

$$\cos(\theta_1-\theta_2) = \frac{|z|^2+|\omega|^2-|z-\omega|^2}{2|z||\omega|}$$



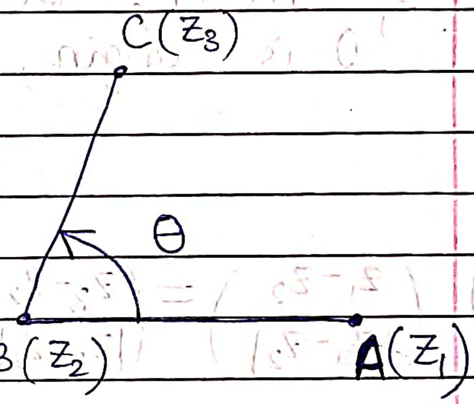
$$\Rightarrow |z|^2+|\omega|^2-2|z||\omega|\cos(\theta_1-\theta_2) = |z-\omega|^2$$

$$\Rightarrow |z-\omega|^2 = (|z|-|\omega|)^2 + 2|z||\omega|\left(1-\cos(\theta_1-\theta_2)\right)$$

$$\Rightarrow \boxed{|z-\omega|^2 \leqslant (|z|-|\omega|)^2 + (\theta_1-\theta_2)^2}$$

Rotation formula

$$\boxed{\left(\frac{z_3-z_2}{|z_3-z_2|}\right) = \left(\frac{z_1-z_2}{|z_1-z_2|}\right)e^{i\theta}}$$



- Angle taken as ↺.

(1) $A(z_1)$, $B(z_2)$, $C(z_3)$ are vertices of ΔABC in ↺ order. If $\angle B = \pi/4$ and $AB = \sqrt{2}\,BC$, then p.t. $(z_2-z_3) = i(z_1-z_3)$

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Q) $A(z_1), B(z_2), C(z_3)$ are vertices of isosceles right angle ΔABC. If $\angle C = \pi/2$ then p.t

$$(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$$

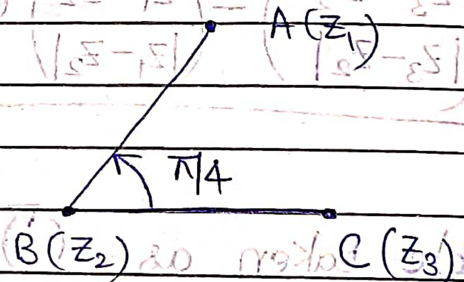
Q) $A(z_1), B(z_2), C(z_3)$ are vertices of ΔABC with $\angle B = \angle C = \frac{1}{2}(\pi - \alpha)$, then p.t

$$(z_2 - z_3)^2 = 4(z_3 - z_1)(z_1 - z_2)\sin^2(\alpha/2)$$

Q) $z^2 - az + b = 0 \begin{matrix} z_1 \\ z_2 \end{matrix}$ s.t. $|z_1| = |z_2|$

If $A(z_1)$, $B(z_2)$ & $\angle AOB = \theta$ where O is origin, p.t. $a^2 = 4b\,c^2_{\theta/2}$

A) $$\left(\frac{z_1 - z_2}{|z_1 - z_2|}\right) = \left(\frac{z_3 - z_2}{|z_3 - z_2|}\right) e^{i\pi/4}$$



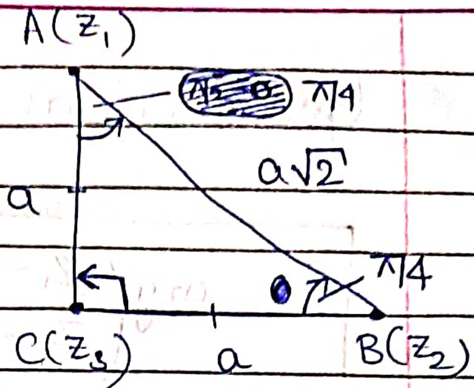
$$\Rightarrow (z_1 - z_2) = (z_3 - z_2)\left(\frac{AB}{BC}\right)\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$

$$\Rightarrow (z_1 - z_2) = (z_3 - z_2)(1 + i)$$

$$\Rightarrow (z_1 - z_3) = (z_3 - z_2)i$$

$$\Rightarrow \boxed{(z_2 - z_3) = (z_1 - z_3)i}$$

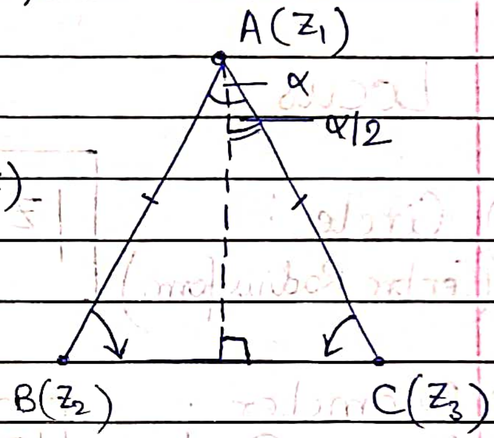
A) B: $\left(\dfrac{z_1-z_2}{a\sqrt{2}}\right)=\left(\dfrac{z_3-z_2}{a}\right)e^{-i\pi/4}$



A: $\left(\dfrac{z_1-z_2}{a\sqrt{2}}\right)=\left(\dfrac{z_1-z_3}{a}\right)e^{i\pi/4}$

$\Rightarrow$ $$\boxed{(z_1-z_2)^2 = 2(z_1-z_3)(z_3-z_2)}$$

A) B: $\left(\dfrac{z_2-z_3}{|z_2-z_3|}\right)=\left(\dfrac{z_2-z_1}{|z_2-z_1|}\right)e^{-i\frac{1}{2}(\pi-\alpha)}$



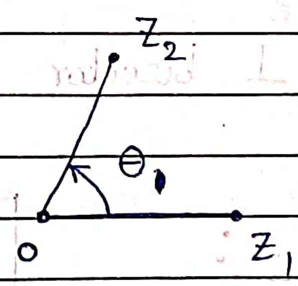
C: $\left(\dfrac{z_2-z_3}{|z_2-z_3|}\right)=\left(\dfrac{z_1-z_3}{|z_1-z_3|}\right)e^{i\frac{1}{2}(\pi-\alpha)}$

Multiplying gives,

$$\frac{(z_2-z_3)^2}{(z_1-z_2)(z_3-z_1)} = \frac{|z_2-z_3|^2}{|z_1-z_2||z_3-z_1|} = \frac{BC^2}{AB\cdot AC} = \cancel{4}\left(\frac{BC}{AB}\right)^2$$

$$= 4\,s^2_{\alpha/2}$$

A) $z_2 = z_1 e^{i\theta}$

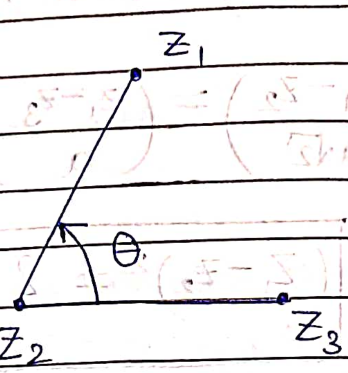


$$\frac{a^2}{b} = \frac{z_1^2(1+e^{i\theta})^2}{z_1^2 e^{i\theta}}$$

$$= (e^{i\theta}+e^{-i\theta})+2 = 2c_\theta+2 \Rightarrow \boxed{a^2 = 4b\,c^2_{\theta/2}}$$

Geometrical Meaning of Arg.

$$\boxed{\theta = \arg\left(\frac{z_1 - z_2}{z_3 - z_2}\right)}$$



Locus

1) Circle : (Centre Radius form) $\boxed{|z - z_1| = r}$ Centre $= z_1$, Radius $= r$

2) Diameter form of ⊙ : $\boxed{|z-z_1|^2 + |z-z_2|^2 = |z_1-z_2|^2}$

Endpts of diameter $= z_1, z_2$

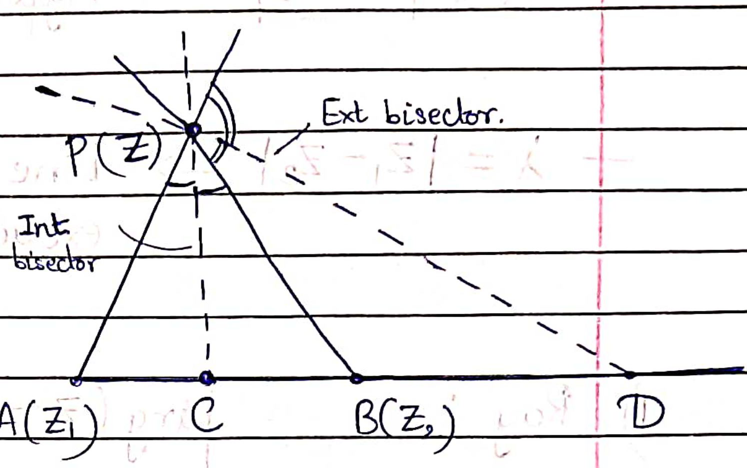
3) ⊥ bisector : $\boxed{|z - z_1| = |z - z_2|}$

is ⊥ bisector of z_1 & z_2.

4) Circle : $\boxed{\left|\frac{z - z_1}{z - z_2}\right| = k}$ where $k \neq 1$.

- If $K>1 \Rightarrow \odot$ contains 'z_2'
- If $K \in (0,1) \Rightarrow \odot$ contains 'z_1'

Proof:



Let C int. divide AB in K : 1 and D ext. divide AB in K : 1.

Let P be pt. on locus. Observe $\frac{PA}{PB} = \frac{AC}{AB} = \frac{K}{1}$

Similarly $\frac{PA}{PB} = \frac{AD}{BD} = \frac{K}{1} \Rightarrow$ PC is int. bisector of $\angle APB$
PD is ext. bisector of $\angle APB$

$\Rightarrow PC \perp PD \Rightarrow$ P subtends $90°$ on CD

$\Rightarrow$ $\boxed{\text{P on } \odot \text{ with diameter CD}}$

5) Ellipse : $\boxed{|z-z_1| + |z-z_2| = \lambda}$ Const.

- $\lambda > |z_1 - z_2| \Rightarrow$ Ellipse with foci z_1 & z_2
- $\lambda = |z_1 - z_2| \Rightarrow$ Line segment with endpts. z_1 & z_2

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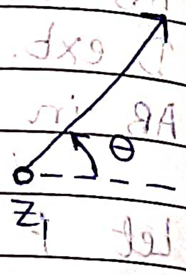
6) Hyperbola : $\boxed{\big||z-z_1|-|z-z_2|\big| = \lambda}$ → Const.

– $\lambda < |z_1 - z_2| \Rightarrow$ Hyperbola with foci z_1 & z_2.

– $\lambda = |z_1 - z_2| \Rightarrow$ Line thru z_1 & z_2 excluding line segment

7) Ray : $\boxed{\arg(z - z_1) = \theta}$

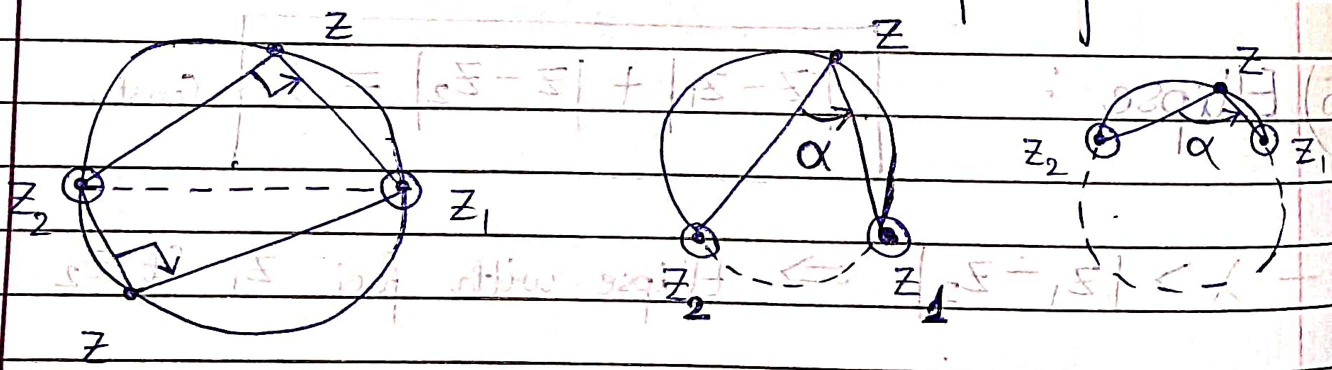


Start pt. $= z_1$

* Ray does NOT contain z_1

8) Arc of ⊙ : $\boxed{\arg\left(\dfrac{z - z_1}{z - z_2}\right) = \theta}$

If $\left|\arg\left(\dfrac{z-z_1}{z-z_2}\right)\right| = \dfrac{\pi}{2}$, we get ⊙ with z_1 & z_2 as endpt. of diameter



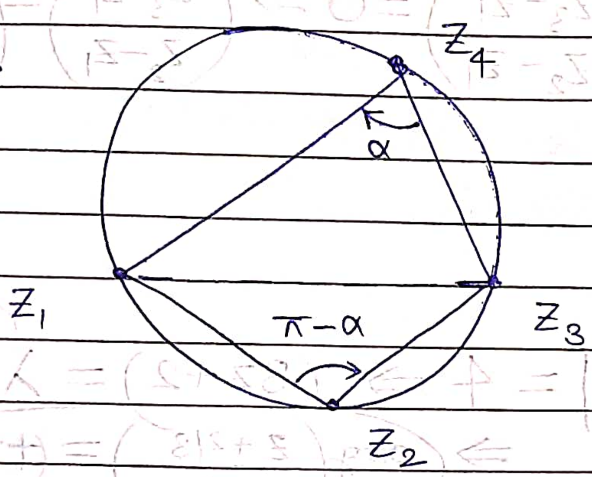
$\alpha < \pi/2$ $\alpha > \pi/2$

* z_1 & z_2 not included

9) Condt. for Concylic pts : z_1, z_2, z_3, z_4 are concyclic

$$\Rightarrow \frac{(z_1 - z_4)}{(z_3 - z_4)} \cdot \frac{(z_3 - z_2)}{(z_1 - z_2)} \text{ is purely real.}$$

Proof :



$$(z_1 - z_4) = (z_3 - z_4) e^{i\alpha}$$

$$(z_3 - z_2) = (z_1 - z_2) e^{i\alpha - i\pi}$$

Multiply to get req. ans.

De Moivre's Theorem

If $n \in \mathbb{Z}$, then, $\left(\cos(\theta) + i\sin(\theta)\right)^{n} = \cos(n\theta) + i\sin(n\theta)$

If $n = \left(\frac{p}{q}\right)$ then, $\left(\cos(\theta) + i\sin(\theta)\right)^{n} = \cos\left(\frac{p\theta}{q} + \frac{2\pi k}{q}\right) + i\sin\left(\frac{p\theta}{q} + \frac{2\pi k}{q}\right)$

rational

where $k \in \{0, \ldots, q-1\}$

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Proof : $\left(\cos(\theta)+i\sin(\theta)\right)^{p/q} = \left(e^{i\theta}\right)^{p/q}$

$= \left(e^{ip\theta}\right)^{1/q} = \left(e^{ip\theta}\ e^{i(2\pi k)}\right)^{1/q}$

$= \left(e^{i(p\theta+2\pi k)}\right)^{1/q} = e^{i\left(\frac{p}{q}\theta + \frac{2\pi k}{q}\right)}$

$= \left(\cos\left(\frac{p\theta}{q}+\frac{2\pi k}{q}\right) + i\sin\left(\frac{p\theta}{q}+\frac{2\pi k}{q}\right)\right)$

'n' th roots of Unity

Solⁿs of $\boxed{x^n=1}$ are called 'n' th roots of unity.

Representation : $1, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}$

where $\boxed{\alpha_k = \cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right)}$

Proof : $\alpha^n = 1 = e^{i(2\pi k)} \Rightarrow \alpha = e^{i\left(\frac{2\pi k}{n}\right)}$

Prop^{t}s —

1) $\{1, \alpha_1, \alpha_2, \ldots, \alpha_{n-1}\} \equiv \{1, \alpha, \alpha^2, \ldots, \alpha^{n-1}\}$

where α is 'n' th root of unity other than 1, ~~with k = odd, if n = even and k = anything, if n = odd~~

$\boxed{\alpha = e^{i(2\pi/n)}}$

2) $1 + \alpha_1 + \alpha_2 + \dots + \alpha_{n-1} = 0$

Proof : $1 + \alpha + \dots + \alpha_{n-1} = 1 + \alpha + \dots + \alpha^{n-1} = \left(\frac{\alpha^n - 1}{\alpha - 1}\right) = 0$

3) $$\alpha_1 \alpha_2 \dots \alpha_{n-1} = \begin{cases} 1 \; ; & n = \text{odd} \\ (-1) \; ; & n = \text{even} \end{cases}$$

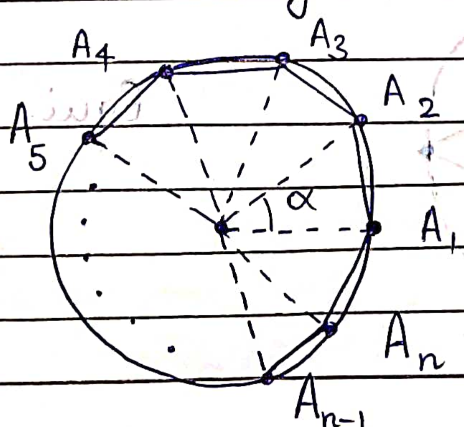
Proof : $\alpha_1 \alpha_2 \dots \alpha_{n-1} = \alpha^{(1+2+\dots+(n-1))} = \alpha^{\frac{n(n-1)}{2}}$

$$= \left[e^{i\left(\frac{2\pi k}{n}\right)}\right]^{\frac{n(n-1)}{2}} = e^{i\pi k(n-1)} = e^{i\pi(n-1)}$$
odd

4) $$\sum_{k=0}^{n-1} \cos(2\pi k/n) = \sum_{k=0}^{n-1} \sin(2\pi k/n) = 0$$

Proof : $\sum \alpha_i = (-1) \Rightarrow \mathrm{Re}\left(\sum \alpha_i\right) = (-1), \; \mathrm{Im}\left(\sum \alpha_i\right) = 0$

5) They represent vertices of regular polyⁿ inscribed in unit circle on argand plane.
One vertex will always be at 1



$\alpha = \left(\frac{2\pi}{n}\right)$

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Cube Roots of Unity

Roots of eqn $\boxed{x^3=1}$

1	ω	ω^2
$1+0i$	$-\frac{1}{2}+\frac{\sqrt{3}}{2}i$	$-\frac{1}{2}-\frac{\sqrt{3}}{2}i$
e^{i0}	$e^{i\pi/3}$	$e^{i(2\pi/3)}$

☆ $\omega^2 = \overline{\omega}$

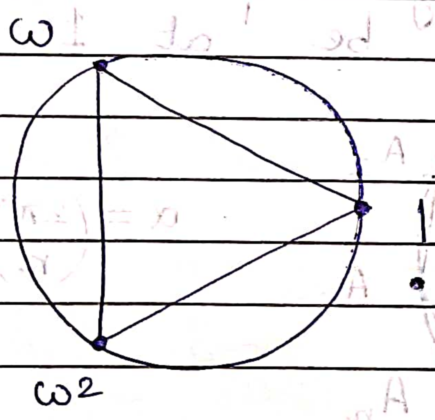
Prop^{t}s :-

1) $1+\omega+\omega^2 = 0$

2) $\omega^3 = (\omega^2)^3 = 1$

3) $1+\omega^n+\omega^{2n} = \begin{cases} 3 \ ; & 3 \mid n \\ 0 \ ; & 3 \nmid n \end{cases}$

4) $\omega^{3n} = 1$
$\omega^{3n+1} = \omega$
$\omega^{3n+2} = \omega^2$

5) 

Equi. Δ.

Q) $\sum_{k=1}^{10}\left(\sin\left(\frac{2\pi k}{11}\right) - i\cos\left(\frac{2\pi k}{11}\right)\right)$

Q) If $1, \alpha_1, \ldots, \alpha_{n-1}$ are 'n' th roots of unity, then p.t.

$$(1-\alpha_1)(1-\alpha_2)\ldots(1-\alpha_{n-1}) = n$$

Q) If $x^3-3x^2+3x+7=0 \begin{smallmatrix}\alpha\\ \beta\\ \gamma\end{smallmatrix}$, p.t. $\sum\left(\frac{\alpha-1}{\beta-1}\right) = 3\omega^2$.

A) (i) $\sum_{k=1}^{10}\left(e^{i\left(\pi-\frac{2\pi k}{11}\right)}\right) = \left[e^{i(9\pi/11)}\right]\left[\sum_{k=1}^{10} e^{i\left(\frac{2\pi}{11}\right)(1-k)}\right][i]$

Let $\alpha = e^{i\left(\frac{2\pi}{11}\right)}$

$\Downarrow$

(i) $= \left[e^{i(9\pi/11)}\right]\left[\sum_{k=0}^{9} e^{-i\left(\frac{2\pi k}{11}\right)}\right][i]$

$= \left[e^{i(9\pi/11)}\right]\left[\sum_{k=0}^{9}\left(e^{i\left(\frac{2\pi k}{11}\right)}\right)\right] = \left[e^{i(9\pi/11)}\right]\left[1+\alpha+\alpha^2+\ldots+\alpha^9\right](i)$

(i) $= \left[e^{i(9\pi/11)}\right]\left[-e^{(-20\pi/11)i}\right] \Rightarrow \boxed{\text{Req.} = i}$

A) $(x-1)(x-\alpha_1)\ldots(x-\alpha_{n-1}) = (x^n-1)$

$\Rightarrow (x-1)[\text{mmm}] + (x-\alpha_1)\ldots(x-\alpha_{n-1}) = nx^{n-1}$

$\Rightarrow \boxed{(1-\alpha_1)\ldots(1-\alpha_{n-1}) = n}$

A) $(x-1)^3 = (-8) \Rightarrow x = (-2)+1, (-2\omega)+1, (-2\omega^2)+1$

$$\Rightarrow \text{Req.} = \left(\frac{-2}{-2\omega}\right) + \left(\frac{-2\omega}{-2\omega^2}\right) + \left(\frac{-2\omega^2}{-2}\right) \Rightarrow \boxed{\text{Req.} = 3\omega^2}$$

A) $$\left(\frac{1}{i}\right)\sum_{k=1}^{10}\left(\cos\left(\frac{2\pi k}{11}\right) + i\sin\left(\frac{2\pi k}{11}\right)\right) = \left(\frac{-1}{i}\right) = i$$

Q) Let A_i $(i = 1, \ldots, n)$ are vertices of regular polyn inscribed in ⊙ with centre $(0,0)$ and radius 1. Then find –

1) $|A_1A_2|\,|A_2A_3| \ldots |A_nA_1|$. 2) $|A_1A_2|^2 + \ldots + |A_nA_1|^2$.

A) $A_k = e^{i\left[\frac{2\pi k}{n}+\alpha\right]}$, $A_{k+1} = e^{i\left[\frac{2\pi k}{n}+\alpha\right] + \frac{2\pi i}{n}}$

1) $$|A_kA_{k+1}| = \left|e^{i\left[\frac{2\pi k}{n}+\alpha\right]}\right|\left|e^{\frac{2\pi i}{n}} - 1\right| = \left|e^{\frac{2\pi i}{n}} - 1\right|$$

$$\Rightarrow \prod |A_kA_{k+1}| = \left|e^{2\pi i/n} - 1\right|^n = \left|e^{\pi i/n}\right|^n \left|2i\left(\frac{e^{\pi i/n} - e^{-\pi i/n}}{2i}\right)\right|$$

$$= \boxed{(2^n)\left|\sin\left(\frac{\pi}{n}\right)\right|^n}$$

2) $\sum\left(|A_K A_{K+1}|^2\right) = n\left[\left|e^{\frac{2\pi i}{n}} - 1\right|^2\right]$

$= n\left|e^{\pi i/n}\right|^2 \left|(2i)\left(\frac{e^{\pi i/n} - e^{-\pi i/n}}{2i}\right)\right|^2$

$= \boxed{4n\left|\sin\left(\frac{\pi}{n}\right)\right|^2}$

Q) Let $x^n = 1 \begin{cases} \alpha_1 \\ \vdots \\ \alpha_n \end{cases}$ Find $\sum_{k=1}^{n-1}\left(|1-\alpha_k|^2\right)$

A) $|1-\alpha_k|^2 = (1-\alpha_k)(1-\overline{\alpha_k}) = 2 - (\alpha_k + \overline{\alpha_k})$

$\Rightarrow \sum\left(|1-\alpha_k|^2\right) = 2(n-1) - \left(\sum\alpha_k\right) - \overline{\left(\sum\alpha_k\right)}$

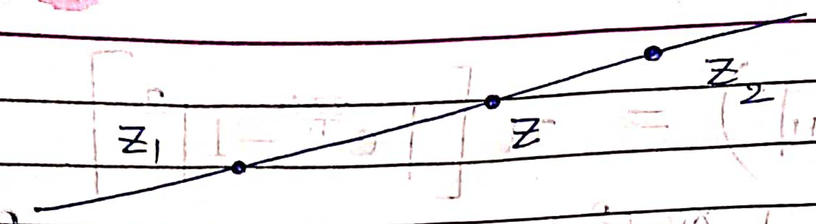
$= 2(n-1) - (-1) - (-1) = 2n$

Eqn of Strt Line

1) Eqn of Strt Line thru $A(z_1)$ & $B(z_2)$.

$$\boxed{\left(\frac{z - z_1}{z_1 - z_2}\right) = \left(\frac{\bar{z} - \bar{z}_1}{\bar{z}_1 - \bar{z}_2}\right)} \quad \text{or} \quad \boxed{\begin{vmatrix} z & \bar{z} & 1 \\ z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \end{vmatrix} = 0}$$

Proof: $(z - z_1) = \lambda(z - z_2)$ & $\arg(\lambda) = 0, \pi$

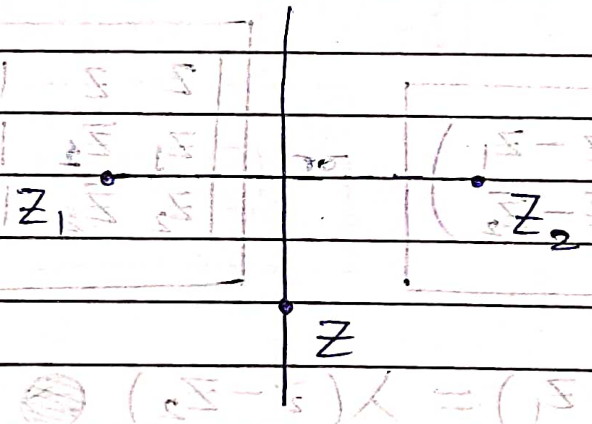


2) General Eq^n of Line:

$$\boxed{\bar{a}z + a\bar{z} + b = 0} \; ; \quad a \in \mathbb{C}, \; b \in \mathbb{R}$$

(Real) Slope of this line $= \boxed{\dfrac{-\operatorname{Re}(a)}{\operatorname{Im}(a)}}$

- Eq^n $\parallel$ to given line : $\boxed{\bar{a}z + a\bar{z} + \lambda = 0}$ real
- Eq^n $\perp$ to give line : $\boxed{\bar{a}z - a\bar{z} + \lambda i = 0}$ real
- $\perp$ dist. of $P(z_c)$ from line : $\boxed{\dfrac{|\bar{a}z_c + a\bar{z}_c + b|}{2|a|}}$
- $\perp$ bisector of $A(z_1)$ & $B(z_2)$: $\boxed{\overline{(z_2 - z_1)}\,z + (z_2 - z_1)\,\bar{z} + \left(|z_1|^2 - |z_2|^2\right) = 0}$



Eq^n of ⊙

1) Centre Radius Form – $\boxed{|z - z_0| = r}$

Centre: z_0, Radius: r

2) General Eq^n of ⊙ – $\boxed{z\bar{z} + a\bar{z} + \bar{a}z + b = 0}$; $a \in \mathbb{C}$, $b \in \mathbb{R}$

Centre $= (-a)$ Radius $= \sqrt{a\bar{a} - b}$

3) Diameter form – $\boxed{(z - z_1)(\bar{z} - \bar{z}_2) + (\bar{z} - \bar{z}_1)(z - z_2) = 0}$

Endpts of diameter : z_1 , z_2